Color-coordinate system from a 13th-century account of rainbows

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We present a new analysis of Robert Grosseteste's account of color in his treatise *De iride* (*On the Rainbow*), dating from the early 13th century. The work explores color within the 3D framework set out in Grosseteste's *De colore* [see J. Opt. Soc. Am. A **29**, A346 (2012)], but now links the axes of variation to observable properties of rainbows. We combine a modern understanding of the physics of rainbows and of human color perception to resolve the linguistic ambiguities of the medieval text and to interpret Grosseteste's key terms.

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1. BACKGROUND

Robert Grosseteste (c. 1175–1253), bishop of Lincoln, was a theologian, scientist, pastor, and politician. As part of a large collaborative research project between medievalists and scientists, we are working to interpret his scientific works. Here we discuss his account of color as it is presented in his short but remarkable treatise, the *De iride* (*On the Rainbow*), from a modern perspective on light and human color vision. Importantly, Grosseteste's reference to the physical phenomenon of rainbows, which will have remained constant over the intervening centuries, allows us to propose an interpretation of the obscure linguistic terms that Grosseteste used consistently in his writing to describe axes of color variation.

The *De iride* is among the last of the scientific works by Robert Grosseteste, dating to the period 1228–1232, a period of his life in which it is easier to establish his career [1]. In 1229/30, he started lecturing in theology to the Franciscans at Oxford, a post he continued in until his elevation to the bishopric of Lincoln in 1235.

The treatise is a sophisticated investigation into the phenomenon of the rainbow. It opens with a long discussion of the science of optics and refraction. The remaining half of the treatise Grosseteste devotes to an investigation of the rainbow itself, how and where it appears, and the shape it is perceived to possess. He concludes with a section on the color variation in rainbows. It is this last section that we consider here. Grosseteste makes substantial use of the theory of color and light expounded in the shorter, and earlier, treatise on color, the *De colore*, which we discuss in detail in an earlier paper [2].

The text was edited by Baur [3, pp. 72–78] and has been translated into English by Lindberg [4]. The treatise has occasioned a body of secondary discussion, including Crombie [5], Eastwood [6], and Boyer [7], especially in light of subsequent medieval discussion of the rainbow, from Roger Bacon, to Dietrich of Freiburg [8, see p. 235 no. 1]. Dales and McEvoy discuss the treatise in their respective chronologies of Grosseteste's works [9,10].

Our forthcoming, related publication [11] presents a new edition of and commentary on the *De iride*, and our present analysis of the text is based on the preparation of that edition and translation. Historical and literary analyses are critical components to understanding the discussion of color in the *De iride*, and its relationship to the *De colore*, but our modern understanding of light, of the physical processes that give rise to rainbows and of human color vision has a great deal more to offer to this question than has hitherto been suspected or explored. To do so is not to admit any form of anachronistic projection of a modern scientific framework onto the achievements of a 13th-century mind, but rather to use all the tools at our disposal to understand the surviving 13th-century text.

2. COLOR IN THE *DE COLORE* AND THE *DE IRIDE*

In the *De colore*, Robert Grosseteste sets out a 3D space of color in which three bipolar qualities, specifically the

Latin pairings—*multa–pauca*, *clara–obscura*, and *purum–impurum*—are used in combination to account for all possible colors [12]. The qualities, *multa–pauca* and *clara–obscura*, are considered as properties of the light, and *purum–impurum* is considered as a property of the "diaphanous medium" in which light is incorporated. According to Grosseteste, whiteness is associated with the triplet *multa–clara–purum*, and blackness with the triplet *pauca–obscura–impurum*. But Grosseteste moves away from the Aristotelian 1D scale of seven colors between white and black, instead defining seven colors close to whiteness, which are generated by diminishing the three bipolar qualities one at a time (to give three different colors), or two at once (to give a further three), or all three at once (to give the seventh). A further seven colors are produced by increasing the qualities from blackness. By allowing infinite degrees of intensification and diminution of the bipolar qualities, he describes a continuous 3D space of color [2].

The appropriate interpretation of the Latin pairings—*multa–pauca*, *clara–obscura*, and *purum–impurum*—that describe the axes of color variation is not well constrained by the context provided in the *De colore*. Grosseteste includes no color terms, nor does he provide examples of objects with diagnostic color. Only in the case of *multa–pauca* does he elaborate slightly by linking *multa* to the intensification of rays by a burning glass. Nevertheless, he claims that combinations of these three bipolar qualities—two qualities of light and one of the medium—permit the generation of all possible colors.

One interpretation of the referential ambiguity of the *De colore* is that Grosseteste had in mind a general perceptual framework for color, which is adaptable to different circumstances of materials and illumination, rather than a definite scheme. In our earlier paper, we considered this interpretation, along with several alternative mappings of his three bipolar qualities onto modern perceptual coordinates [see 13 for a comprehensive account of modern color spaces], including cylindrical systems of hue-saturation-value (e.g., HSV or HSL) and Cartesian systems of primaries (e.g., RGB or LMS). Each mapping presented some advantage but also suffered from inconsistencies with the logic of the text.

Although the *De colore* presents the modern reader with an unresolvable puzzle, Grosseteste, in the *De iride*, provides us with a clue: the variation of color in rainbows. With reference to Grosseteste's discussion of color in the *De iride*, we are now able to make firmer links between Grosseteste's bipolar axes and perceptual color space; in particular, we ask whether the axes that Grosseteste identified span perceptual space effectively. There are two components to this work: one of them about Grosseteste's color theory, the other showing how different rainbows plot in human color space. The two components are intimately linked because the detailed physical modeling of different types of rainbow, which are directly inspired from the observations and comparisons set out in the *De iride*, allows us to test the hypothesis that the color variations exhibited by rainbows span perceptual color space in a way that is consistent with the abstract description in the *De colore*.

In the *De iride*, the section on color starts with a recapitulation of the framework that was laid down in the *De colore*. Again color is inherently associated with the interaction of light and materials: "… color is luminosity [14] mixed with [15] a diaphanous medium" [the translation here and in the following is based on our forthcoming critical edition of the Latin text, 11]. Variation in color results from variation in the qualities of the light and the medium: the "diaphanous medium is differentiated according to purity and impurity [*puritatem et impuritatem*], the luminosity is divided four ways; that is, according to brightness and dimness [*claritatem et obscuritatem*] and then according to copiousness and scarcity [*multitudinem et paucitatem*], and the generation and the diversity of all colors occurs according to the combinations of these six distinguishing characteristics."

In the *De iride*, Grosseteste goes beyond this abstract conceptualization of color to link these axes of variation to properties of rainbows. He writes, "The variety of color in the different parts of one and the same rainbow occurs chiefly because of the copiousness and scarcity [*multitudinem et paucitatem*] of the solar rays. For where there is a greater multiplication of rays, the color appears clearer and more luminous; and where there is a smaller multiplication of rays, the color appears dim and close to purple" [16]. And later, "In fact, the difference in the colors between one rainbow and another arises sometimes from the purity and impurity [*puritate et impuritate*] of the recipient diaphanous medium, sometimes from the brightness and dimness [*claritate et obscuritate*] of the luminosity impressed on it. For if the diaphanous medium is pure [*purum*] and the luminosity is bright [*clarum*], the color of the rainbow will be more similar to white and light. But if the recipient diaphanous medium should contain a mixture of smoky vapors and the brightness [*claritas*] of the luminosity is scarce, as occurs around sunrise and sunset, the color of the rainbow will be less brilliant and more obfuscated."

This passage in the *De iride* therefore provides the potential link from Grosseteste's terminology to physically repeatable phenomena. One of his bipolar axes is assigned to different parts of the rainbow [*multitudinem et paucitatem*], another to the quality of the diaphanous medium giving rise to different rainbows [*puritate et impuritate*], and a third to the luminosity of the incipient light [*claritate et obscuritate*]. In the following, we explore the path suggested by this association to see whether it resolves the impasse of a modern reading of the *De colore* alone.

3. GENERATION OF COLOR IN RAINBOWS

We now turn to the physical processes that give rise to color in rainbows. According to the basic scheme, described by Descartes [17] and Boyer [18], sunlight enters a raindrop and is reflected internally one or more times before finally exiting. A single internal reflection gives rise to the primary bow, and two internal reflections generate the secondary bow. If we follow a bundle of parallel rays from the sun entering the droplet and undergoing one internal reflection, their exit paths are dispersed but many are concentrated along one particular angle, the caustic ray. In this scheme, based on geometric optics, the caustic angle depends only upon the refractive index of the droplet, which determines the angular deviation of the ray at each air-water interface. Descartes was able to use this account to predict the location of the rainbow, but was missing an explanation of rainbow colors, which relied on Newton's observation (see [19] for discussion) that different wavelengths of light, associated with different colors, have different refractive indices in a given medium, so the spectral content of the exiting light varies as a function of angle to produce the familiar colors of the rainbow.

Predictions from geometric optics do not, however, capture two important physical characteristics of rainbows: the existence of supernumerary arcs, and the dependence of rainbow colors on droplet size. Using wave theory, Airy developed an excellent approximation of the primary rainbow [20]. A rigorous model of all of the scattering processes caused by a spherical droplet of water, such as external reflection, multiple internal reflections, surface waves, and diffraction is provided by Mie theory. A reformulation of Mie theory, known as the Debye series [21], also provides an exact solution, but it allows the separation of contributions due to specific types of scattering (e.g., those involving a specific number of internal reflections).

4. METHODS: MODEL SIMULATIONS

The simulations we present here were obtained using the MiePlot program (MiePlot v4, available at www.philiplaven.com/mieplot.htm) [22,23]. Our simulations show the $p = 2$ term of the Debye series (i.e., for light that has undergone a single internal reflection), with a light source corresponding to the spectral energy distribution of sunlight [24] and an apparent angular diameter of 0.5°, interacting with spherical droplets of water.

Figure 1 shows the relative energy of light as a function of scattering angle, interacting with water droplets with radius $r = 200$ μm, for a set of 16 monochromatic lights (from 400 to 700 nm in 20 nm steps), and their sum. Each wavelength shows the peak intensity at a characteristic angle, together with additional peaks corresponding to the supernumerary arcs. The combination of light across wavelengths indicates the visible spectrum of the resulting rainbow, which changes in dominant wavelength [25] as a function of scattering angle. The dispersion of each monochromatic band over a range of scattering angles (caused primarily by the variation in angles of incidence at the air–water interface for rays within a parallel beam meeting a spherical droplet, and additionally by further scattering phenomena) means that the rainbow spectrum is not a pure spectrum, but is instead desaturated by the superposition of different wavelengths. There is also a strong variation in total intensity as a function of angle.

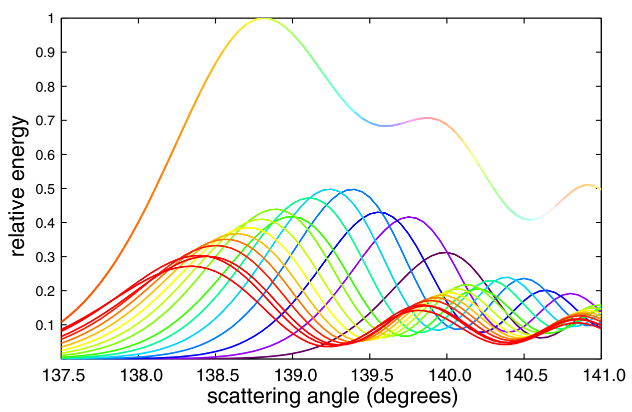

Fig. 1. Plot of relative energy for scattering by a spherical droplet of water of radius $r = 200$ μm as a function of scattering angle for monochromatic sources (from 400 to 700 nm in steps of 20 nm producing 16 curves), and their sum (arbitrarily scaled by 0.25 for plotting). Curves are pseudo-colored to specify wavelength and the resultant color of the mixture.

Scattering phenomena can produce complex variations in spectral intensity distributions. Even for a monochromatic light source, the prediction is nontrivial since the scattering is dependent on the size of the scattering sphere. In his seminal paper, Lee [24] developed diagrams (now known as "Lee diagrams") to illustrate how the appearance of rainbows varies with the size of the scattering water droplets.

These are the tools we need to tease out the consequences of Grosseteste's two separate comparisons in the *De iride*: the variation in color that occurs within a rainbow, and the variation that occurs between different rainbows due to the quality of the "diaphanous medium." To quantify these variations, we have modeled the difference in colors within a rainbow parameterized by scattering angle, and assumed that the most significant source of the difference in colors between rainbows is the size of the raindrops.

Figure 2 is a modified Lee diagram showing the pseudocolor representation of the spectrum of light obtained at a range of scattering angles (between 137.5° and 141°) and a range of droplet radii (between 200 and 1000 μm). Linking this to Grosseteste's characterization of color in rainbows, variation between rainbows corresponds to moving parallel to the

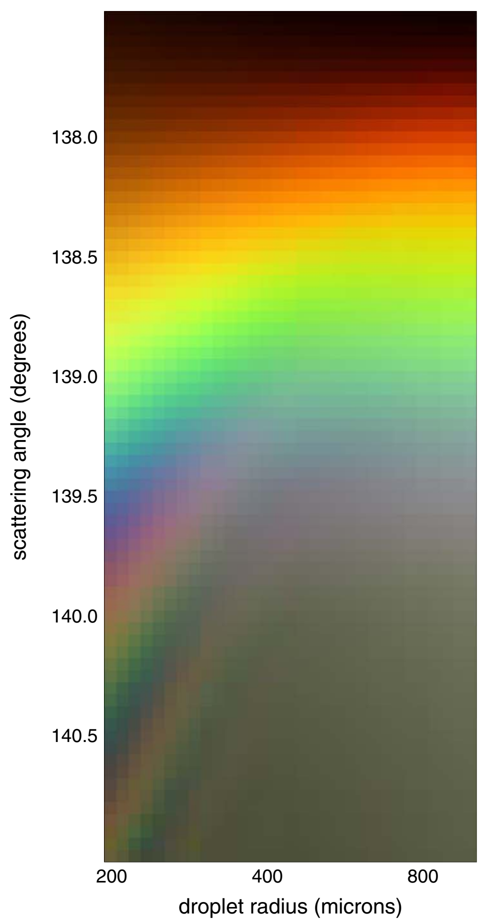

Fig. 2. Pseudo-color representation of the spectra obtained as a function of scattering angle (within a rainbow) and droplet radius (between rainbows).

abscissa and variation within a rainbow corresponds to moving parallel to the ordinate.

There are clearly several factors that can parametrically influence the appearance of different rainbows. Grosseteste identifies cases where "the diaphanous medium should contain a mixture of smoky vapors," which is consistent with the appearance of mist or fog in which the droplet size is small, but he additionally notes cases where "the brightness [*claritas*] of the luminosity is scarce, as occurs around sunrise and sunset." The spectrum of sunlight that impinges on water droplets to produce a rainbow will depend on the amount of atmosphere the light has encountered, which can be conveniently parameterized by the total air mass along the solar ray, which has a one-to-one relationship to solar elevation angle under particular atmospheric conditions [26]. To model the spectral effects of solar elevation angle, we start with high-accuracy measurements of the solar flux per unit wavelength above the atmosphere (air mass $= 0$) (from the standard overhead sunlight spectrum from the Hubble Space Telescope calibration database) modified by the three processes of (1) molecular Rayleigh scattering [27, p. 126]; (2) aerosol scattering [27, p. 126 with the parameter chosen to correspond to normally clear conditions]; and (3) ozone Chappuis-band absorption [28 with the parameter chosen to correspond to 3 mm of ozone per airmass at standard temperature and pressure (STP), and the absorption cross-section taken from the website of the Institute of Environmental Physics (IUP) at the University of Bremen: http://www.iup.physik.uni-bremen.de/gruppen/molspec/index.html]. Each of these processes depends on air mass and therefore on solar elevation angle. We neglect the effects of molecular oxygen and water vapor absorption, which have rather little effect within the visual spectral range we consider.

Different incident spectra reweight the relative wavelength composition of the rainbow. Small changes in the assumed spectrum of sunlight do not alter our conclusions, since we are predominantly interested in *changes* of the spectrum according to scattering angle and droplet size, and on the effects of *changes* imposed on the sunlight spectrum by atmospheric factors. It is also clear that the bow's background can have a substantial effect on the appearance of the bow [29]. We have not explicitly modeled these additional effects, in part because it is difficult to do so, but primarily because they are not mentioned explicitly in the text of the *De iride*.

5. RESULTS: RAINBOW SPECTRA IN HUMAN COLOR SPACE

Presented with the linguistic, combinatorial account of color in the *De colore*, we might ask how to interpret the bipolar qualities that Grosseteste uses to navigate color space. Since human color vision relies on the signals from just three spectrally selective classes of cone photoreceptors, human color spaces are limited to three dimensions of variation, based on the signals in the cones or on transformations of those signals. Here we ask how the interpretation of Grosseteste's linguistic terms might be facilitated by his association of these terms with color variation in natural rainbows, and whether the axes of variation that he identifies can be said to effectively span the perceptual space of color.

A naïve interpretation of the difference of color *within* a rainbow is that it corresponds simply to a variation in hue. Indeed it is common in nontechnical accounts of the rainbow to link rainbow colors to spectral colors. Differences *between* rainbows might simply correspond to changes in saturation. At very small droplet sizes (e.g., for $r = 10$ μm), the rainbow or fogbow is colorless and perfectly desaturated. Under these assumptions, the axis *multa–pauca* should correspond to hue, and changes in saturation should be carried by *purum–impurum* (and perhaps *clara–obscura*). However, this sits uncomfortably with other assertions in the *De colore*. For Grosseteste, the points attained by maximal and minimal excursions along the three axes of variation are assumed to be whiteness and blackness. In the hue-saturation-value coordinate system of human color perception, lines of constant hue are radial while lines of constant saturation are concentric.

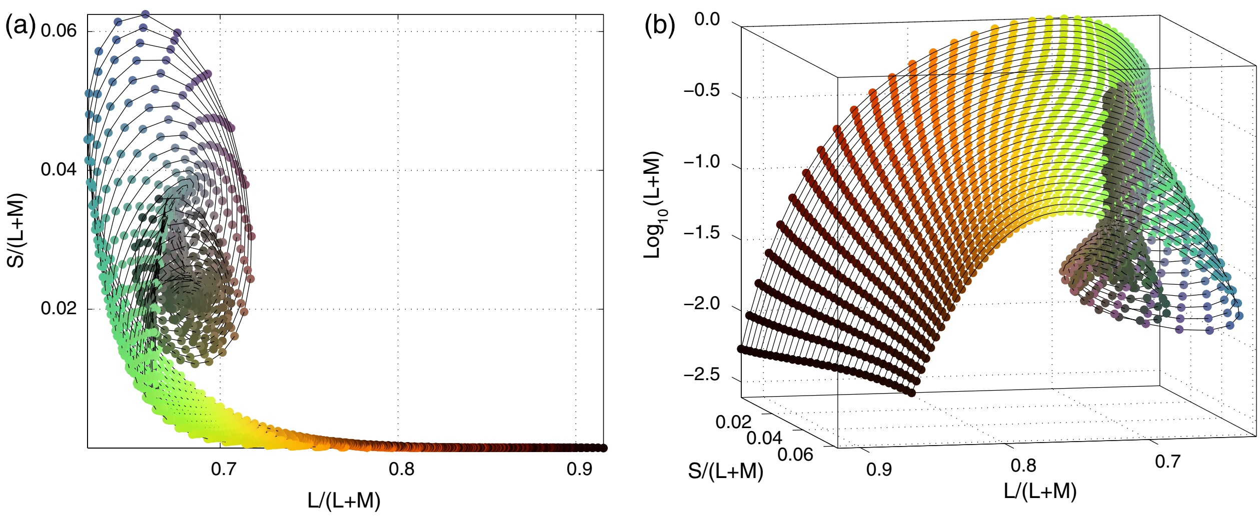

Fig. 3. Plots of the MacLeod–Boynton coordinates of spectra obtained by simulating scattering of light by spherical droplets of water, as occurs in natural rainbows. Each pseudo-colored symbol represents a particular combination of scattering angle and droplet size, according to the range represented in the Lee diagram of Fig. 2. Black lines are used to link points that share the same droplet size and are therefore characterized as within the same rainbow. (a) A projection onto the chromatic plane (Media 1). (b) An oblique projection, additionally showing the variation in (log) luminance (Media 1).

Furthermore, Grosseteste explicitly associates *multa–pauca* with the intensification of rays by a burning glass.

By plotting the family of spectra represented in the Lee diagram in a 3D space that describes human trichromatic color vision, we can characterize the way in which these variations span the 3D gamut of possible colors. We include two representations of our simulated rainbow spectra that are relevant for human color vision. Figure 3 uses a cone-excitation space with the MacLeod–Boynton axes [30] spanning the equiluminant plane and a logarithmic luminance axis. In this representation, total solar flux is arbitrarily scaled, but the relative positions within the diagram are direct consequences of physical changes in the spectra. Figure 4 shows our simulated rainbow spectra in CIE 1976 $L*$, $a*$, $b*$ (CIELAB) space, which is approximately perceptually uniform such that equal distances within the space correspond approximately to equally perceived differences in color. CIELAB space was originally intended for surface colors and not self-luminous sources, and it requires specification of a reference white-point of a particular luminosity. We have chosen to set the white-point to the maximum luminosity within each single rainbow (on the grounds that these colors will be available simultaneously to the observer, whereas colors from different rainbows will not).

Colored points are included for each pairing of scattering angle and droplet radius included in the Lee diagram of Fig. 2. Thin black lines join colors obtained for different scattering angles at a constant droplet size (i.e., within a rainbow); the organization of colored symbols can be traced to map out the transformations imposed by changing droplet size at a constant scattering angle (i.e., between rainbows).

The gamut is limited compared to the full spectral locus [see 4(c) and 4(d)], but nevertheless it provides reasonable coverage of the chromaticity plane. The locus of rainbow colors forms a surface in trichromatic human color space, and a striking feature is the spiraling nature of that surface. This effect can be understood from the intensity plot in Fig. 1, as only the spectra that are long-wavelength dominated at low scattering angles are relatively pure (saturated), while spectra at high scattering angles are produced from progressively

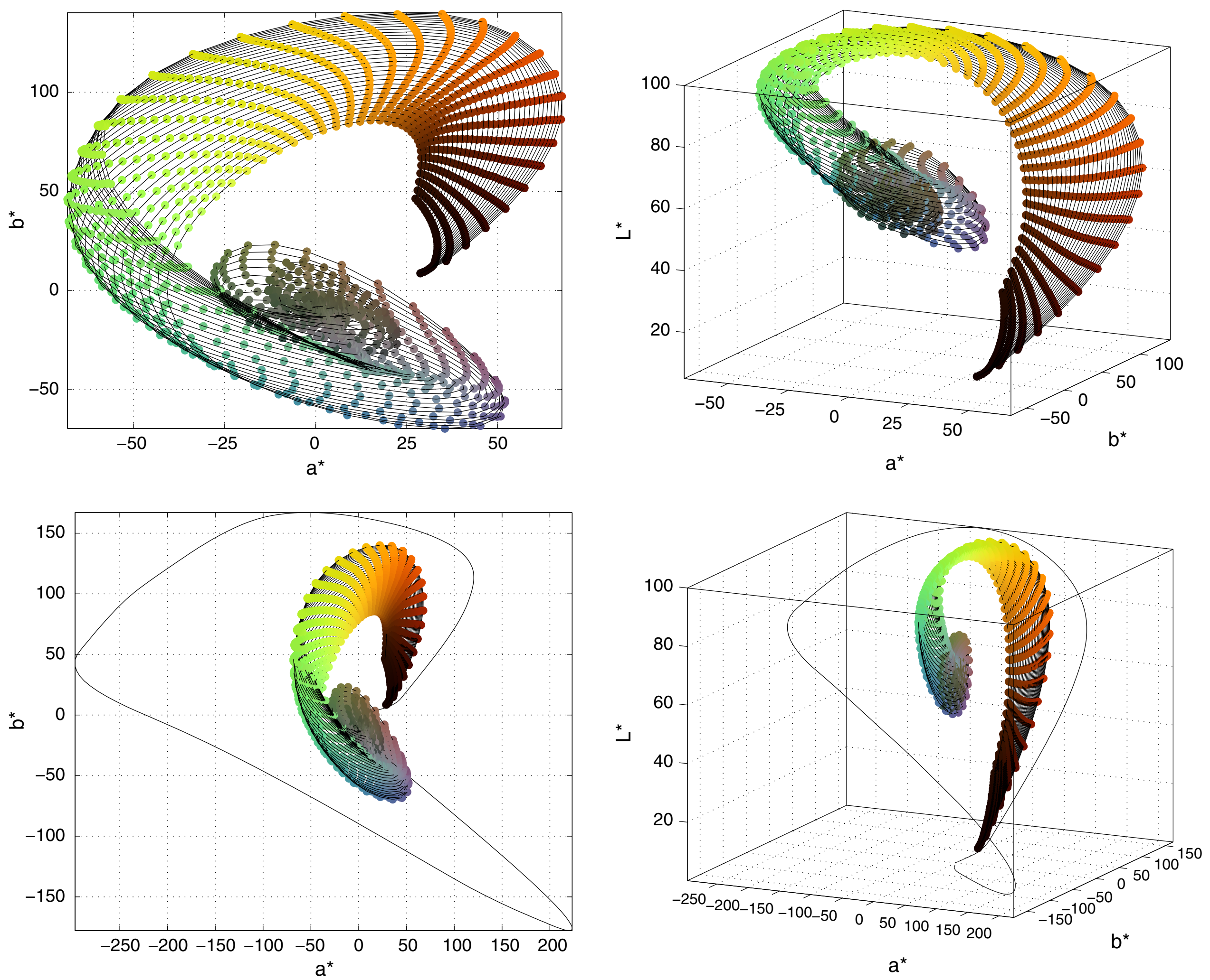

Fig. 4. Plots of the CIELAB coordinates of spectra obtained by simulating scattering of light by spherical droplets of water, as occurs in natural rainbows. Each pseudo-colored symbol represents a particular combination of scattering angle and droplet size, according to the range represented in the Lee diagram of Fig. 2. Black lines are used to link points that share the same droplet size and are therefore characterized as within the same rainbow. The white-point is set to correspond to the daylight illuminant D65 at the maximum luminosity available within each rainbow. (a) A projection onto the chromatic plane (Media 2). (b) An oblique projection, additionally showing the variation in luminance (Media 2). (c) Analogous to (a), but including the spectral locus. (d) Analogous to (b) but including the spectral locus.

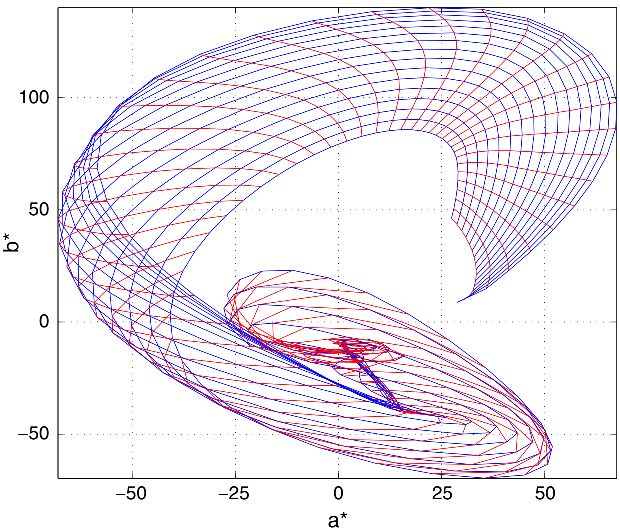

Fig. 5. Plot of the CIELAB ($a*, b*$) coordinates of spectra obtained by simulating scattering of light by spherical droplet of water, as occurs in natural rainbows (Media 3). The white-point is set to correspond to the daylight illuminant D65 at the maximum luminosity available within each rainbow. A sparse grid of points is connected by lines with constant scattering angle (red) and constant droplet size (blue). These two sets constitute a possible coordinate system for the perceptual subspace spanned by possible rainbows generated from the unmodified solar spectrum.

more overlapping monochromatic spectra and are thereby desaturated. In the MacLeod–Boynton chromaticity diagram and in CIELAB space, this corresponds to a spiraling progression, from close to the spectrum locus toward the white-point in the center of the diagram. Coupled with this chromatic variation, the colors within a rainbow show a strong variation in luminance, from low luminance at low scattering angles to much higher values for mid-scattering angles before declining again for the desaturated spectra obtained at high scattering angles.

The progressions obtained by changing droplet size generate a second set of spiraling loci, less tightly wound and intersecting the original set. Together these parameters span a 2D surface.

The modeling here includes only a single droplet size within each rainbow. It is clear, however, that in natural systems a range of droplet sizes will be encountered. Making the reasonable assumption of no interaction between droplets; the effects of a range of droplet sizes can be modeled as a weighted sum of the spectra for each constituent droplet-size in the distribution. It is clear from the plots that this mixing will simply convert the 1D locus of colors within a single rainbow (i.e., one of the thin black lines in the plot) to a 2D band that is smeared along the surface defined by the variation in droplet size.

6. SPIRAL COORDINATE SYSTEMS FOR PERCEPTUAL COLOR SPACES

The mapping of the Lee diagrams onto the three dimensions of perceptual CIELAB space immediately suggests another approach to covering color space with a coordinate system. Removing the colored points from the projection of the rainbow surface onto the chromatic plane [Fig. 4(a)], and connecting equal scattering angles and equal droplet sizes with two different sets of curves (colored red and blue) makes explicit a new coordinate system for the surface, shown in Fig. 5.

Spiral coordinate systems are not novel, and the Cartesian (e.g., RGB) and cylindrical (e.g., HSV, HSL) coordinate systems discussed in our previous paper [2] do not exhaust the set of natural orthogonal coordinate systems that partition a finite connected space; there are not only independent systems, but orthogonal coordinate systems that are spiral in configuration. Confining discussion initially to a 2D space (e.g., of a^* and b^* in the chromatic plane), we recall that any complex transformation of the Cartesian plane will

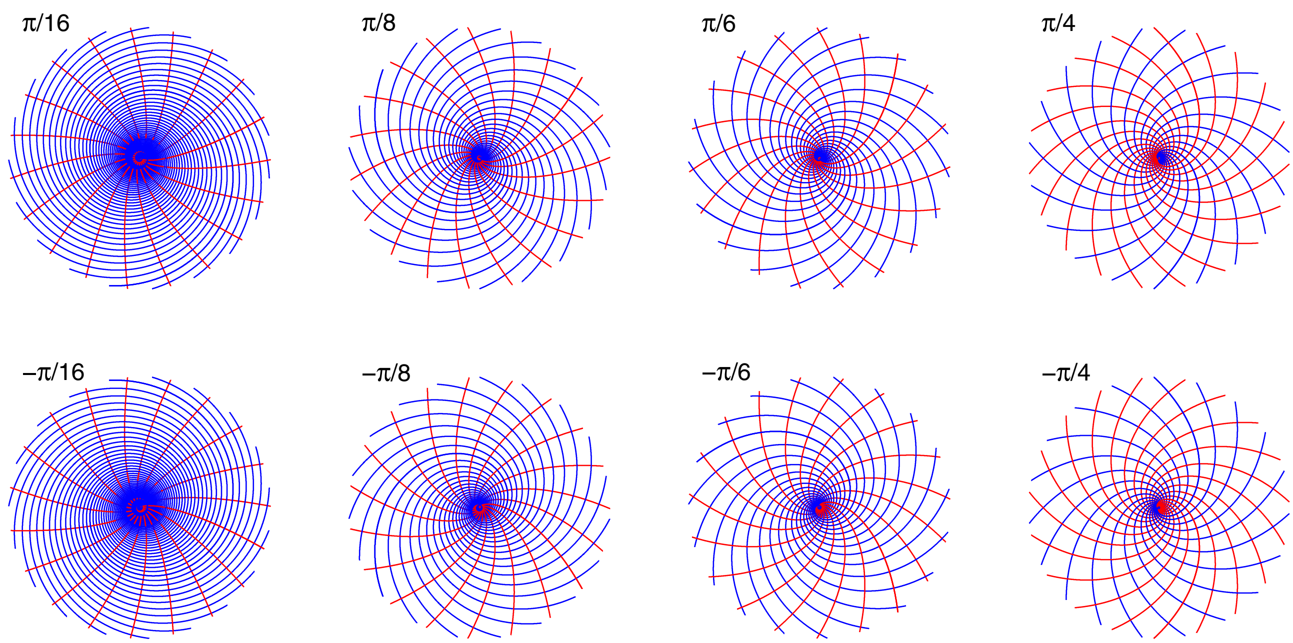

Fig. 6. Examples of "logarithmic-polar" coordinate systems generated from Eqs. (1) and (2) with various values of the tilt angle, ϕ. A value of $\phi = \pi/4$ generates symmetric sets; decreasing values result in one set becoming tighter, the other looser, until the purely radial-circumferential system emerges in the limit of $\phi = 0$. A sign change in ϕ generates a change in the handedness of the system.

generate an orthogonal set of coordinates. In particular, the logarithmic-polar transformation,

$$x + iy = e^{\rho + i\theta}, \tag{1}$$

transforms the orthogonal grid rotated by an angle of ϕ to the (x, y) plane,

$$\begin{aligned} y &= y_1 + (\tan \times \phi)x \\ y &= y_{-1} - (\cot \times \times \phi)x, \end{aligned} \tag{2}$$

into a set of intersecting spirals, each parameterized by values of the constants y_1 and y_{-1}. The angle of tilt ϕ "tunes" the coordinate system so that one of the two sets of spirals is more or less radial, and the other more or less circumferential (see Fig. 6, where the twin intersecting sets of spiral coordinate grids are given for a range of ϕ).

A significant property of this set of coordinate systems (where ϕ is finite) is that the central point lies at one extremity of *both* coordinates. It is only in the limit of the purely radial-circumferential system at $\phi = 0$ that this property is lost. To take the illustrative example of the case that the mapped space is the chromatic plane of hue and saturation, any logarithmic-polar coordinate system with $\phi \neq 0$ permits the neutral (white) point to be the source of both coordinates. The standard radial and circumferential system of hue and saturation themselves corresponds to the case of $\phi = 0$.

We recall that one challenge to finding a satisfactory mapping of Grosseteste's bipolar color qualities, as defined in the *De colore*, onto modern perceptual descriptions was precisely that "whiteness" is explicitly at one extremity of all three axes (associated with the triplet, *multa–clara–purum*). This is true of the Cartesian RGB description of color space, but that interpretation was ruled out by the difficulty of mapping any of the three qualities onto either primary or secondary hues. In the *De iride*, Grosseteste gives us an essential clue to unlocking the meaning of his terms. Comparing Figs. 5 and 6, we find that his association of two color qualities onto types of rainbow and positions within rainbows does indeed map

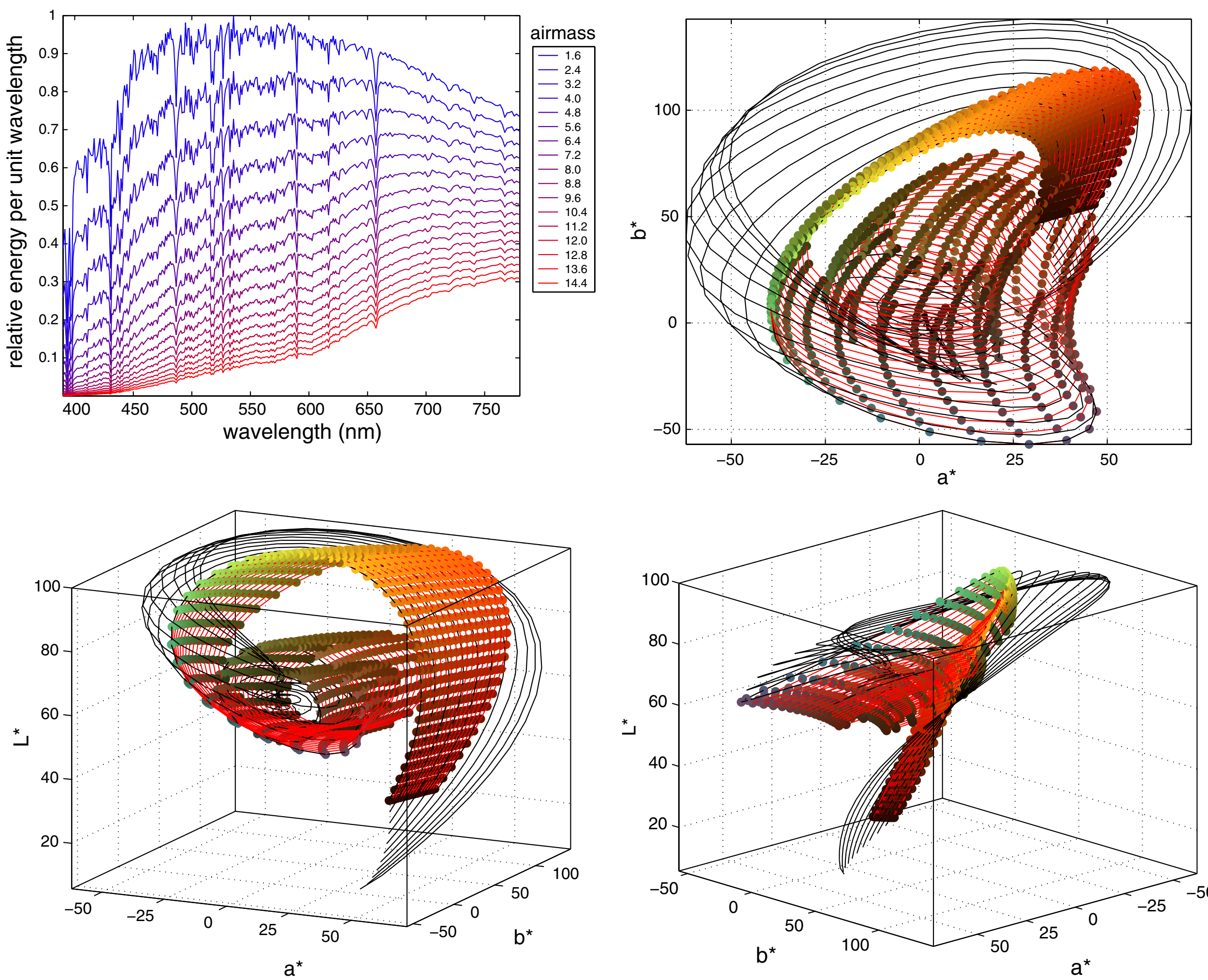

Fig. 7. Effect of solar elevation angle on the solar spectrum and consequently on the rainbow locus. (a) The family of spectra (energy per unit wavelength) obtained with air mass values from 1.6 to 14.4 in steps of 0.8 (corresponding to solar elevation angles from 38.6° to 3.2° [26]) and ozone and aerosol factors of 1, based on the extinction model using molecular and aerosol scattering [27] and ozone absorption [28]. (b)–(d) The set of rainbow loci in CIELAB space that are produced by using these as the incident solar spectra (Media 4). The black lines give a skeleton outline of the surface that is obtained as a function of droplet size for the highest solar elevation. The red lines and colored symbols locate the surface that is obtained as a function of air mass for the smallest droplet size. The white-point is set to correspond to the daylight illuminant D65 at the maximum luminosity available within each rainbow.

a significant portion of the chromatic plane with a spiral coordinate system, albeit of generalized form. The effective tilt angle ϕ is not constant throughout the plane (passing through zero within the red hues), and the existence of the supernumerary arcs creates a reversal of the ordering of saturation between bows of different droplet size (causing a multivalue when collapsed onto the chromatic plane). Nor is the rainbow-spiral coordinate system orthogonal at every point like the ideal logarithmic-polar system. However, it does share the same topology, and, essentially, the property of mapping a plane with independent coordinates, both of which originate from the origin.

Extension to a mapping of the full 3D space is straightforward and most easily achieved (in the case of the ideal log-polar system) by extending the spiral coverage of the plane into a third dimension in the same way that cylindrical coordinates are produced from polar coordinates in the plane. Employing intensity as the third dimension to span the perceptual color space naturally produces white and black as two opposite poles of the cylindrically extended log-polar coordinate system.

An extension of the rainbow-spiral space is subtler. From Figs. 4(a) and 4(b) Media 2, we have seen that rainbows illuminated with solar spectra are already strongly distorted in the luminance coordinate, with high values of L^* corresponding to intermediate scattering angles. Although translating the rainbow spirals parallel to the L^* direction provides a reasonable coverage of the color solid, we can see from Fig. 4(b) (and in greater 3D clarity from the movie) that a third axis, which correlates increasing L^* with a shift within the chromatic plane, is in general more orthogonal to the rainbow surfaces.

Grosseteste is explicit in the *De iride* that the third bipolar quality of color, characterized by the *claritas* or *obscuritas* of the luminosity impressed on a rainbow, depends upon solar elevation angle. The dominant spectral consequence of solar elevation angle is a reduction in overall intensity and a relative loss of short wavelengths, causing the familiar reddening and darkening observed at sunrise and sunset. Figure 7(a) shows the family of spectra obtained with air mass values from 1.6 to 14.4 (corresponding to solar elevation angles from 38.6° to 3.2°, which are well within the range that will produce visible rainbows) and ozone and aerosol factors of 1. Figure 7(b) shows the set of rainbow loci in CIELAB space that are produced by using these as the incident solar spectra. The black lines give a skeleton outline of the surface that is obtained as a function of droplet size for the highest solar elevation. The red lines and colored symbols locate the surface that is obtained as a function of air mass for the smallest droplet size. From this plot, and particularly from the viewpoint rotation available in Media 4, we note that the two surfaces describe separable variations in color. Again, the white-point is set within a rainbow, so this figure de-emphasizes the strong variation in overall intensity produced by changes in solar elevation angle.

7. CONCLUSION

Generalizations of cylindrically extended log-polar coordinate systems do possess the properties of abstract axes of color space described in Robert Grosseteste's 13th century treatise on color, the *De colore*. Without further information and evidence, there would be no incentive to push this interpretation further. But the identification of his coordinates with perceptual properties of natural rainbows, from the final section of his work on rainbows, the *De iride*, provides the motivation to explore further. Mapping the full physical optics of rainbow generation for bows of different droplet size generates just such a coordinate system for color space. Plotted in MacLeod–Boynton or CIELAB space, the spiral coordinates arising from these two axes of variation, *multa–pauca* associated with scattering angle and *purum–impurum* associated with droplet size, are locally distorted and display a variation in tilt angle when projected onto the chromatic plane. They are also additionally twisted into the third coordinate of luminance. While variation in the qualities of light and medium identified by Grosseteste does not cover the full gamut of real lights bounded by the spectral locus, their capacity to span the central region of the chromatic plane is striking. The further allowance of variation in the "luminosity impressed on" the medium, *clara–obscura* associated with the sun's elevation, provides a means of sweeping the 2D surface spanned by rainbow spirals through the color solid.

There are, of course, some imperfections in the proposed scheme. From the *De colore*, we have the triplet of [*multa*, *purum*, *clara*] as whiteness and [*pauca*, *impurum*, *obscura*] as blackness, with *multa* additionally associated with concentration of rays by a burning glass. We also know, both from the *De colore* and from the *De iride*, that *multa–pauca* and *clara–obscura* are properties of the light, while *purum–impurum* has to do with the medium. A spiral coordinate system allows two axes, *multa–pauca* and *purum–impurum*, to terminate at the white point. But there are loose ends. In the *De iride*, *impurum* (a component of blackness) is associated with smoky vapors, but vapor or mist (small droplet sizes) would desaturate colors, making them closer to white. Similarly, *pauca* (also a component of blackness) is associated with purple, which is the least-saturated region of the rainbow locus. There are also limitations to our modeling. Natural rainbows usually have much smaller color gamuts than those depicted here because of additive mixing with background light [29], which we have not modeled. Our simulation considers only spherical droplets, though it is known that nonspherical droplets influence rainbow formation [31], and although we consider the effects of aerosol scattering and ozone on the solar spectrum, we have not parametrically investigated the effects of variation in aerosol and ozone concentrations. Nevertheless, the simulations presented here indicate that the three components of variation identified in the *De iride*, within a rainbow (parameterized by scattering angle) and between rainbows (parameterized by droplet size and the effect of air mass on solar spectrum), can be used to navigate perceptual color space reasonably effectively.

This analysis provides another example of how modern methods within the scientific fields descendent, in some verifiable measure, from thinking in the medieval period can illuminate the questions, assumptions, and goals of scientific writing then. The absence of explicit discussion of color terms in the *De colore*, coupled with the primacy of the "rainbow coordinates" in the *De iride*, is surprising to modern eyes, since to us spectral hue is a dominant descriptor of color. But, as we observe, the common identification of rainbow colors with spectral hues is misleading. A modern observer projects familiarity with dominant wavelengths and hue onto discussion of rainbows. For Grosseteste, in the 13th century, the colors of the rainbow were better described in terms of the "copiousness" and "clarity" of light and the "purity" of the medium (*multa–clara–purum*), which cross and intersect our current coordinates of hue, saturation, and brightness, but do not align with them. In the same way that modern coordinate systems to navigate color space can derive from manipulations of color that we experience (in the RGB color cube for example), so it seems possible that to a medieval eye the structure of the color palette arises from nature's primary phenomenon that generates a variation across perceptual color space, the rainbow.

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15. Similarly, he uses here *admixtum cum*, not *incorporatum* as he did in the *De colore*, which we translate as "mixed with," not "embodied in."
16. Grosseteste uses here the Latin adjective *hyazinthinus*, from the substantive *hyacinthus*, which we choose to translate as purple. The sources here are complex and are based on medieval references to gem stones and other color terminology. So, the identification with any particular color is blurred, but on-balance we believe that violet or purple with some red is an appropriate interpretation. The fact that in the (perceptual) hue circle (but not on a wavelength scale) violet, purple and red are adjacent is also worth noting.
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